

CS7351 传感系统原理与设计

Principles and Design of Sensing Systems

Sensors – Inertial Measurement Unit (惯性传感器)

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2025年秋季



Paper Presentation Schedule

Name	Topic	Title
朱腾杰		
吴媚		
谢婧		
付晨龙		
何弈拓		
张卜元		
RICHARD DANIEL LIU		
邱宇豪		
王唯逸		
郑颜昊		
Distribution of topics: IMUx2, Acoustic, Camera, Securityx2, Navigation, Magnetic, Miscx2		

Presentation requirement:

1. Please plan a **15-20 min presentation** and a **5 min Q&A**
2. The presentation **should include**:
 - Motivation, related works, technical challenges, solution, evaluation, and future works
3. The presentation should be prepared English but can be presented in either Chinese or English

Paper list-CS7351-2025

<https://docs.qq.com/sheet/DUnZaaVh5QWJVSUNm?tab=BB08J2>

Please go through the paper list and put your selection here. Please complete the form **by October 9th**.



Why would you consider reproduce a work?

- Practicing artifact evaluation for your future works
- It may not be that hard after all
 - CS works are supposed to have high reproducibility
 - **After carefully reading the paper,** check out online resources, even talk to the authors!
- You will receive bonus points for reproducing the work!



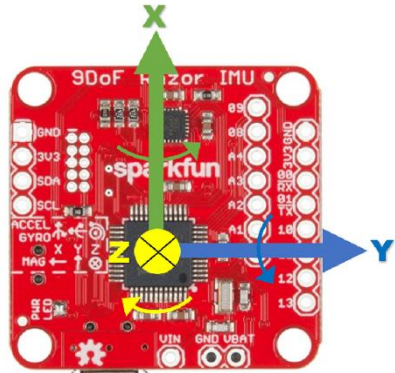


Quick Recap: How to Design Your Sensing Application?

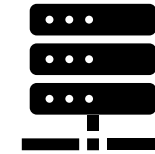
- Step 1: What problem you want to solve?
- Step 2: Examine the 4 key design principles



4 Key Principles for Designing Sensing Systems



Sensing Modalities



Computation



Power and Energy



Connectivity



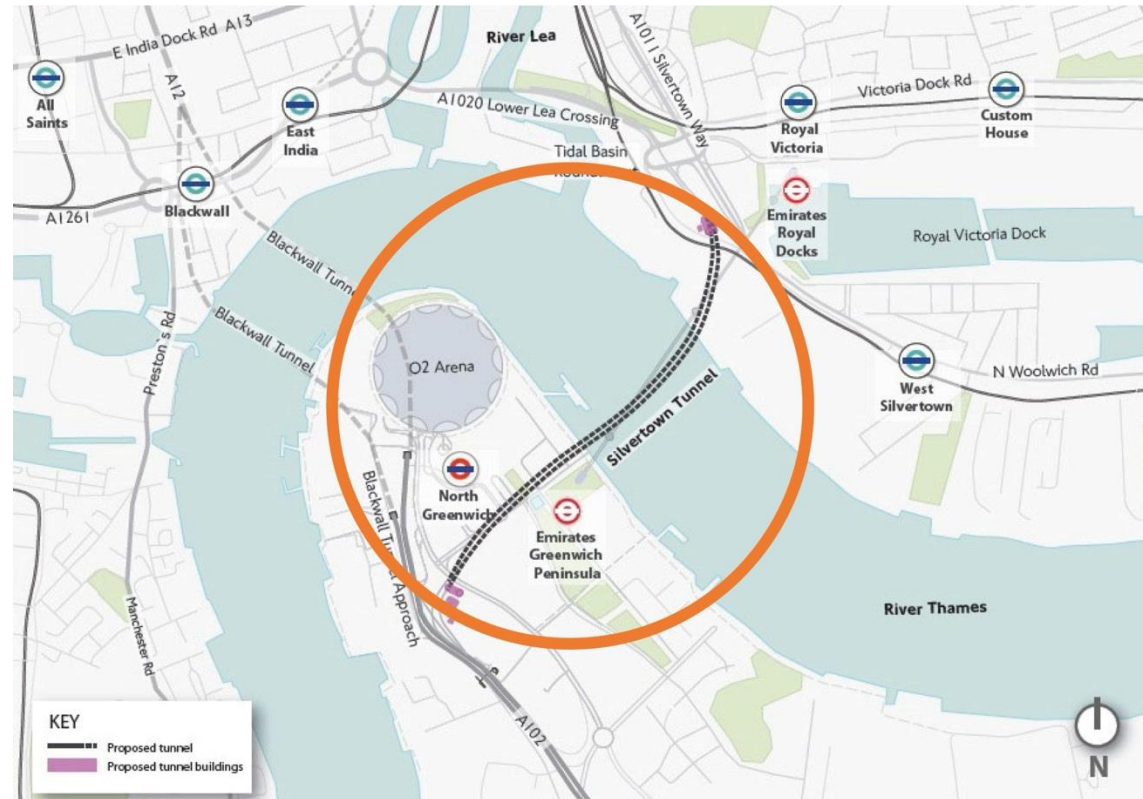
Outline

- An Introduction of IMU
- IMU Error Model
- IMU Calibration



Example Application of IMU: Inertial Navigation

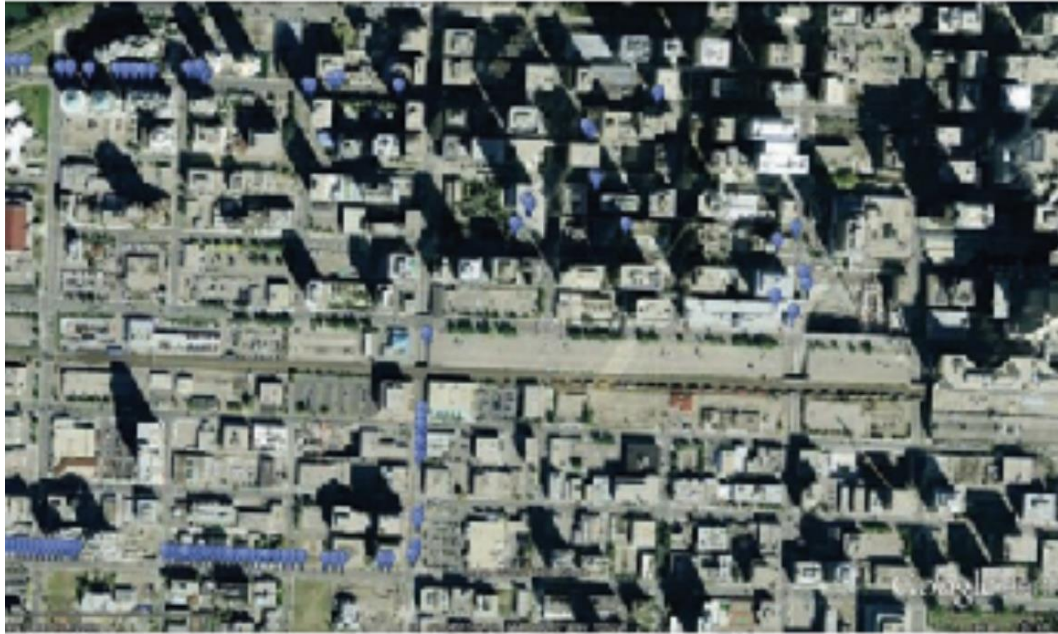
- Navigate in tunnels



- How to navigate when you cannot receive satellite data?



Example Application of IMU: Mitigate Urban Canyon Problem



GPS only

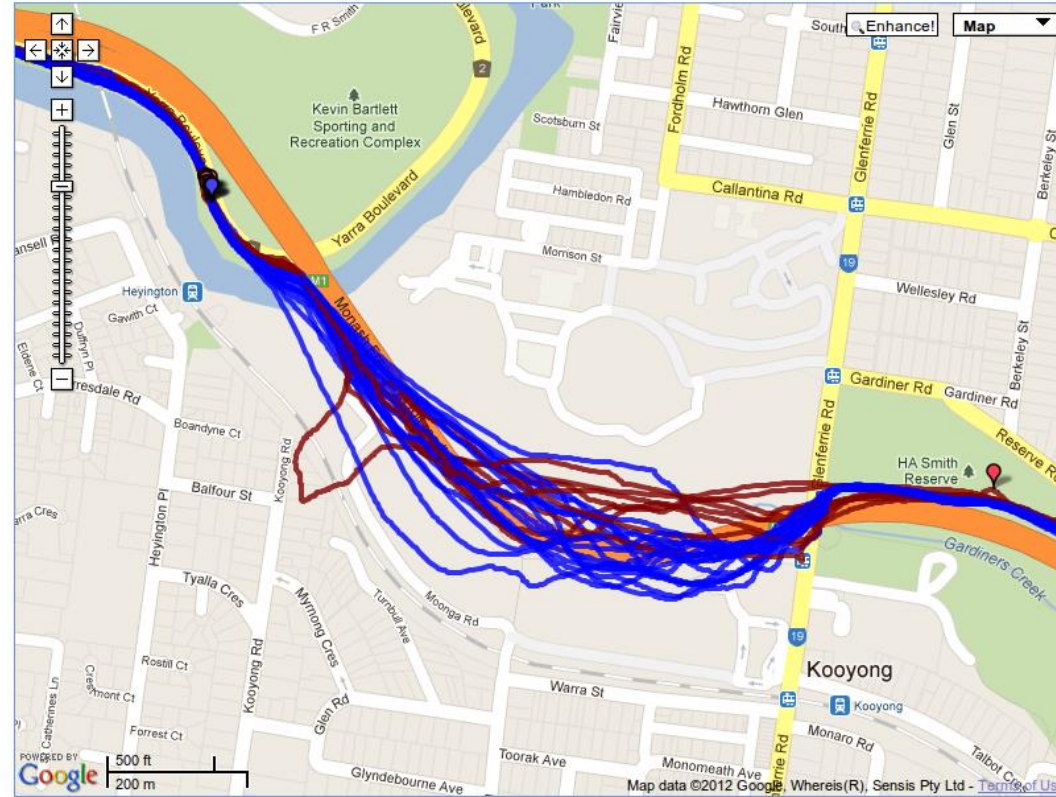


GPS+INS

- Key idea: integrate acceleration data over time to derive location



Is Inertial Navigation Accurate?

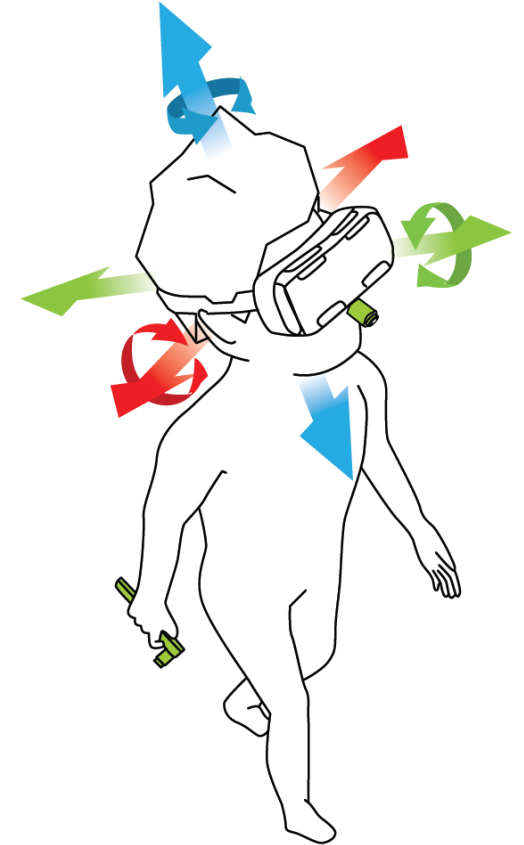


- How to analyze the error?



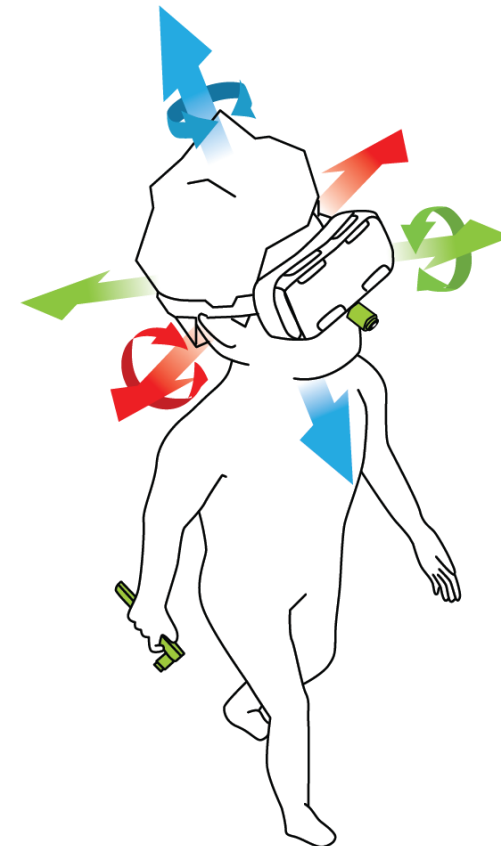
Example Application of IMU: Headset Tracking

- **Goal:** track location and orientation of head or other device
- **Coordinates:** Six degrees of freedom:
 - Cartesian frame of reference (x,y,z)
 - Rotations represented by Euler angles (yaw, pitch, roll)





- **Gyroscope** measures angular velocity ω in degrees/s
- **Accelerometer** measures linear acceleration a in m/s^2
- **Magnetometer** measures magnetic field strength m in μT (micro-Teslas).





Where are IMUs Used Today?





Outline

- An Introduction of IMU
- IMU Error Model
- IMU Calibration



Why it is Called Inertial Measurement Unit?

- What is inertia?
 - It means an object will continue its current motion until some force causes its speed or direction to change.
- Rotation inertia
 - A rotating rigid body maintains its state of uniform rotational motion.

Inertia Example #1: Why you need to wear a seatbelt (especially if you are a giraffe)

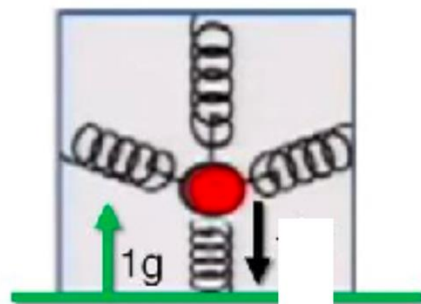
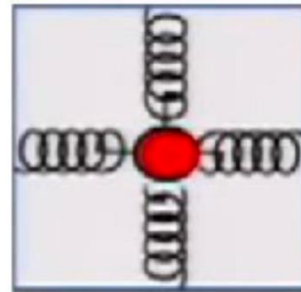




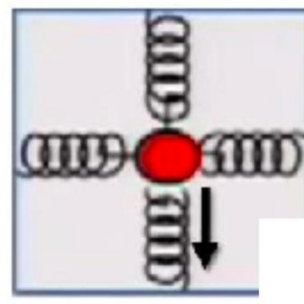
How Accelerometer Work?

- It measures acceleration

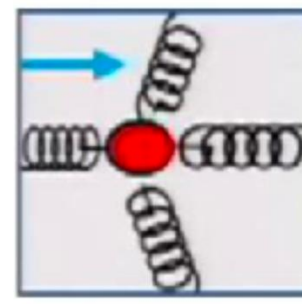
Mass on spring



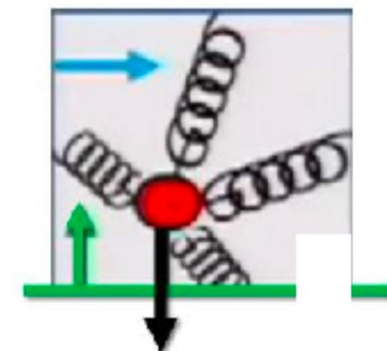
Gravity
 $1g = 9.8\text{m/s}^2$



Free Fall



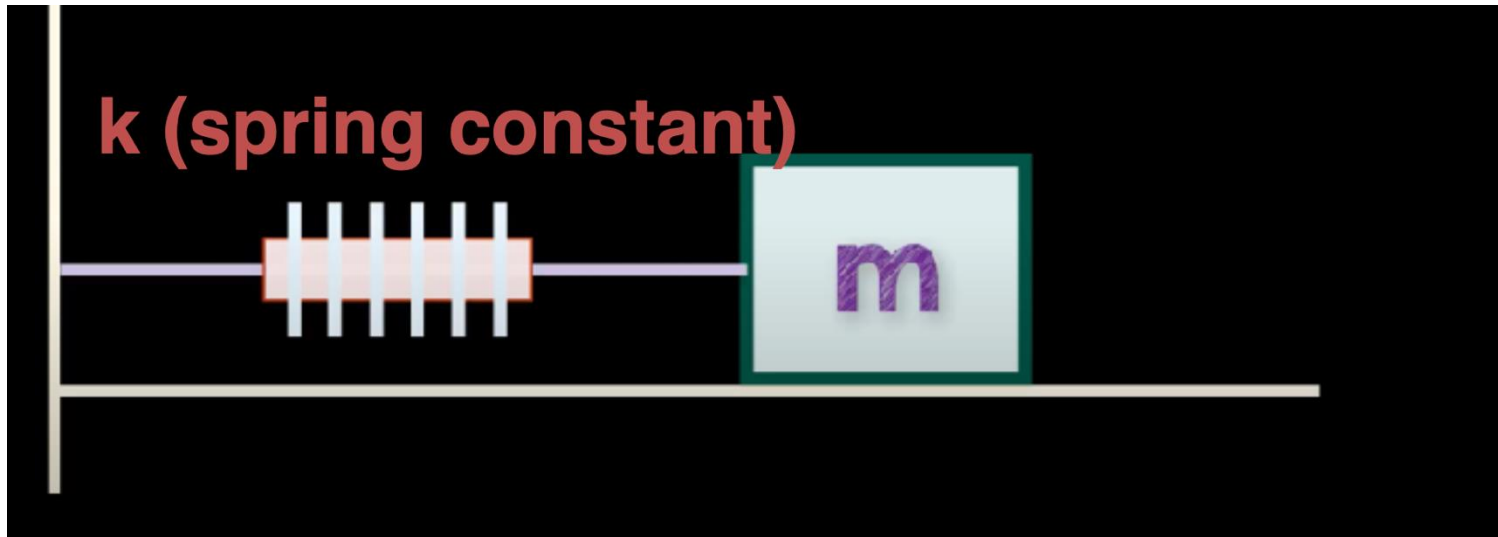
Linear Acceleration



Linear
Acceleration
plus gravity



How Accelerometer Work



Hooke's Law

$$F = kx$$

Newton's Law

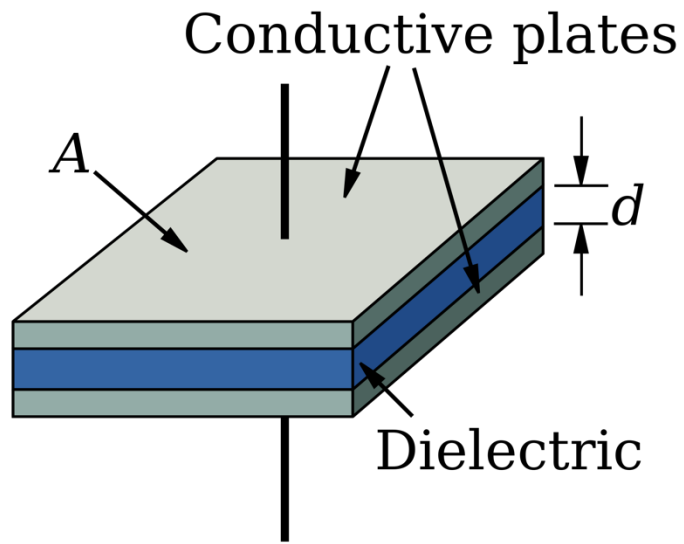
$$F = ma$$

$$\Rightarrow a = \frac{k}{m}x$$

- How to measure x ?



How Accelerometer Work



$$C = \frac{\epsilon A}{d}$$

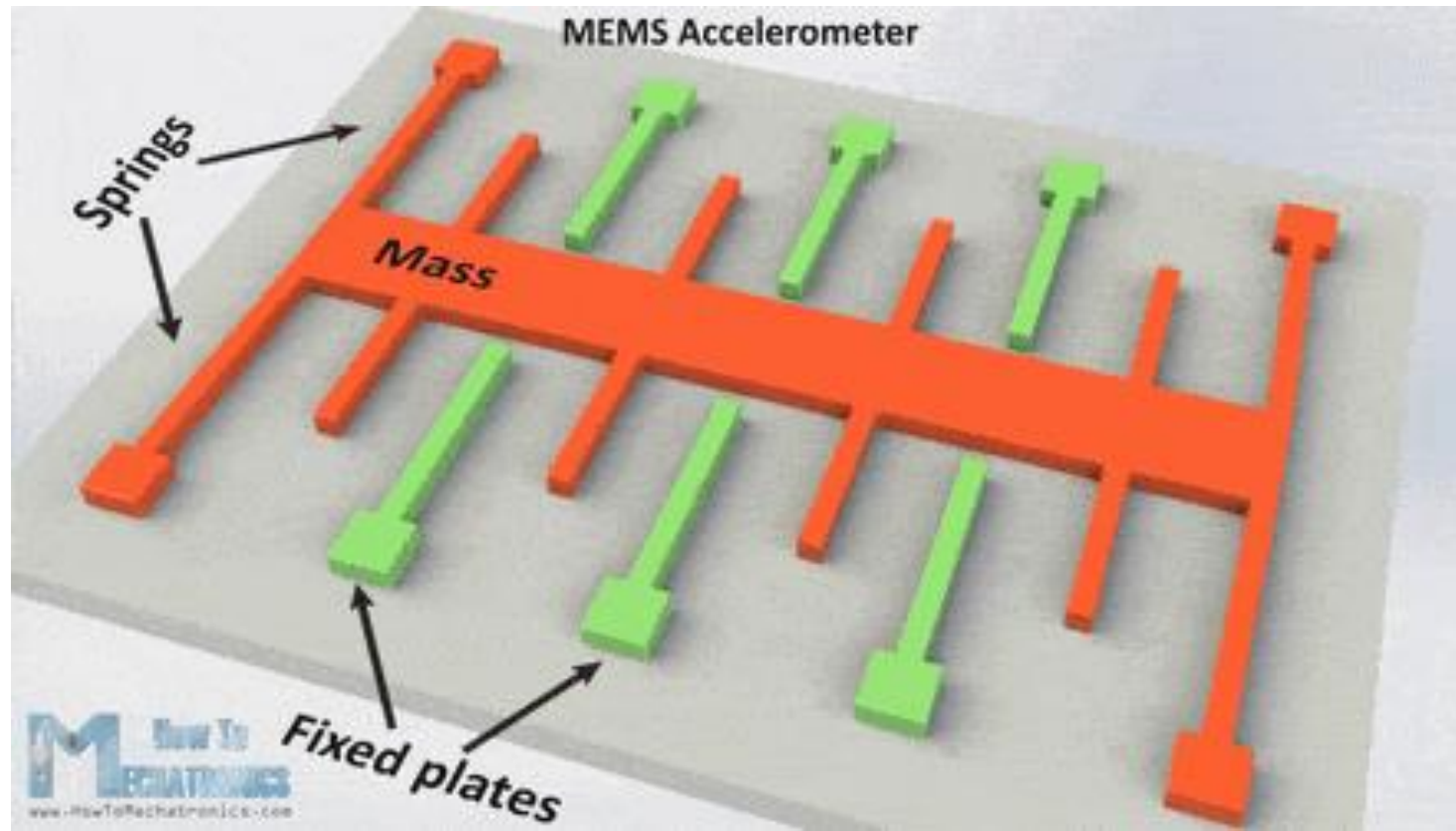
- C: capacitance
- A: area
- ϵ : permittivity
- d: distance

We can measure the varying displacement to measure acceleration!



How to Measure the Changing Distance?

- Most common approach is to use MEMS (Micro electro-mechanical systems)

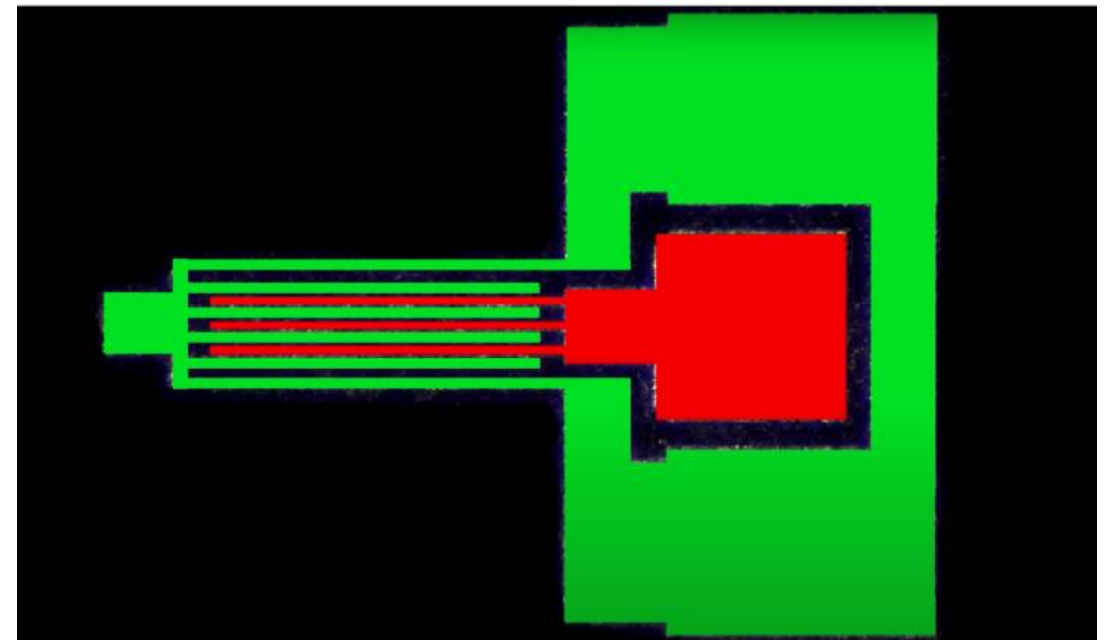
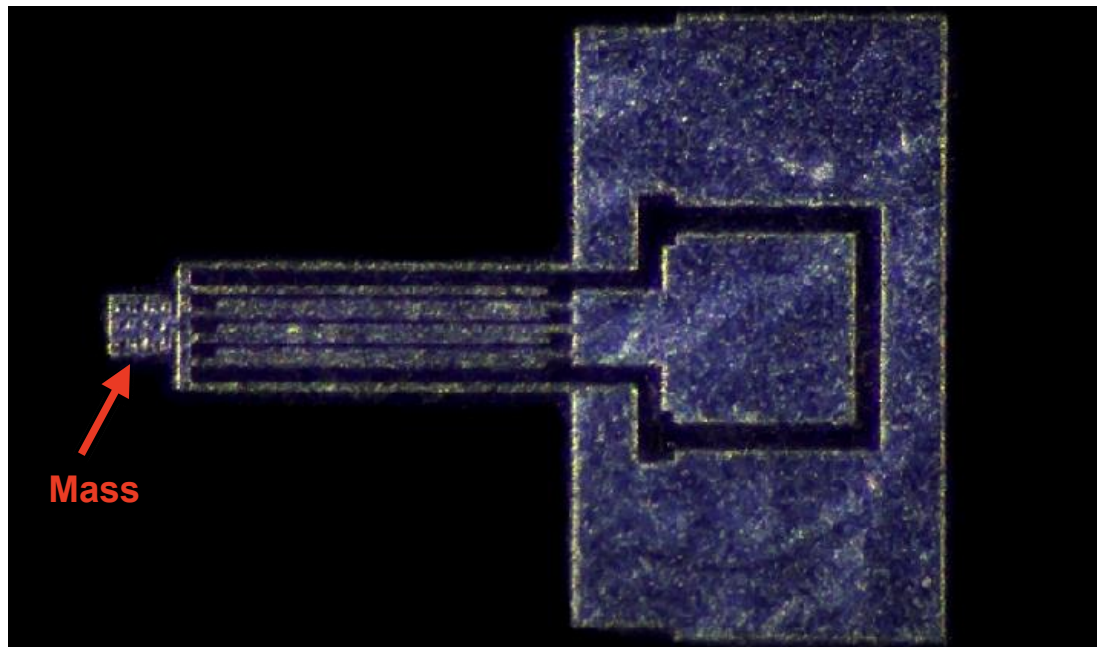




How to Measure the Changing Distance?

- Most common approach is to use MEMS (Micro electro-mechanical systems)

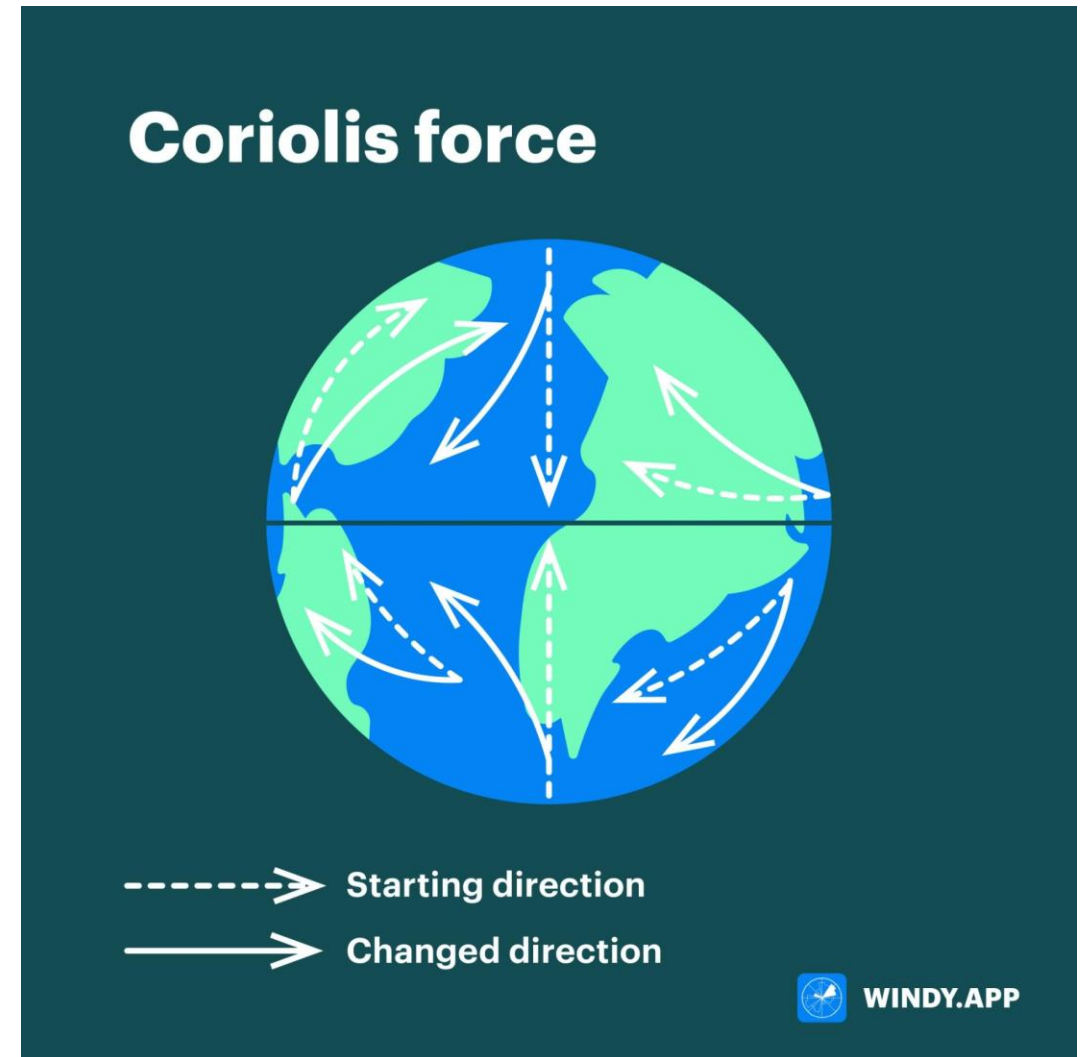
MEMS Accelerometer





How Gyroscope Work

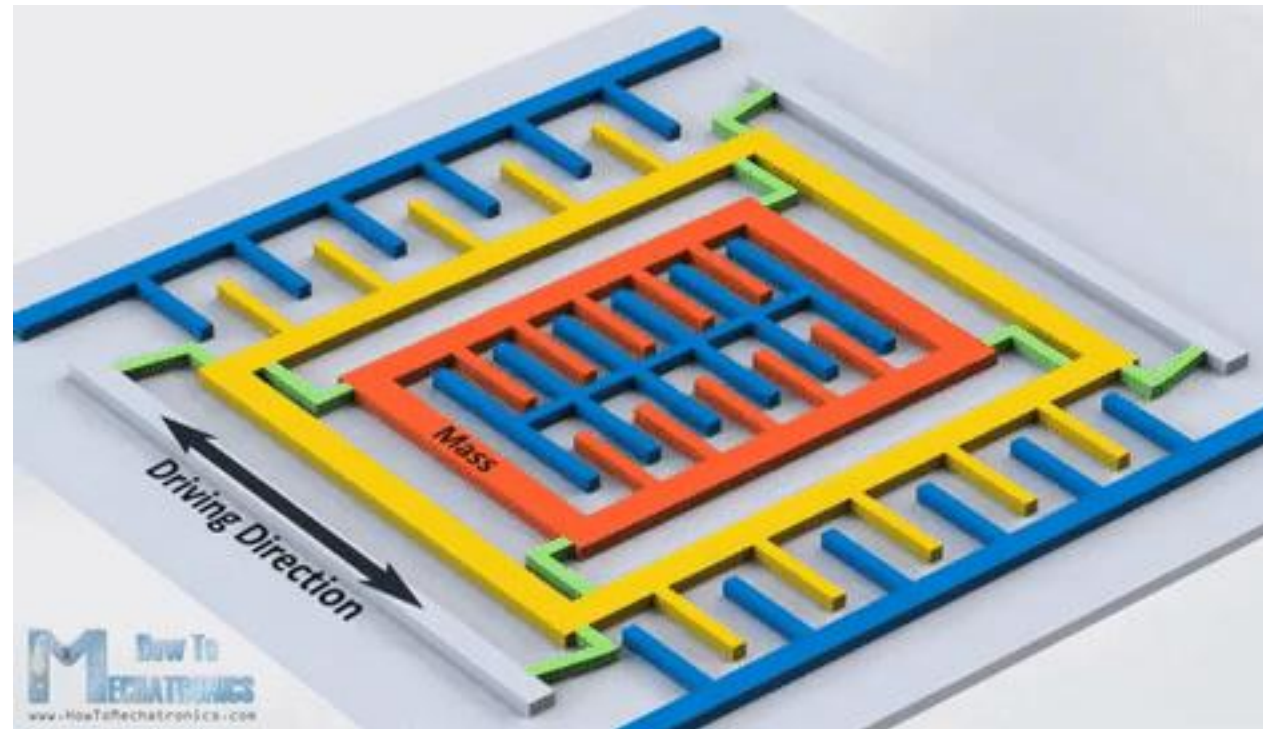
- It measures angular speed (rad/s)
- Foundation: Coriolis Effect





How Gyroscope Work

- It measures angular speed (rad/s)
- Foundation: Coriolis Effect + MEMS



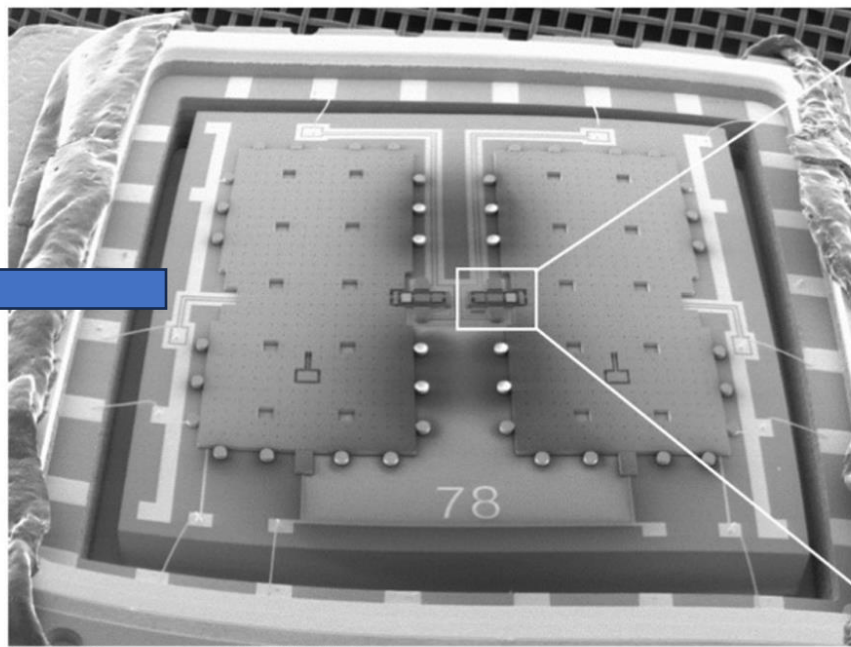


IMU原理&&结构

Accelerometer sensing element inside the chip

左图：Accelerometer sensing element
扫描电子显微镜（SEM）图像

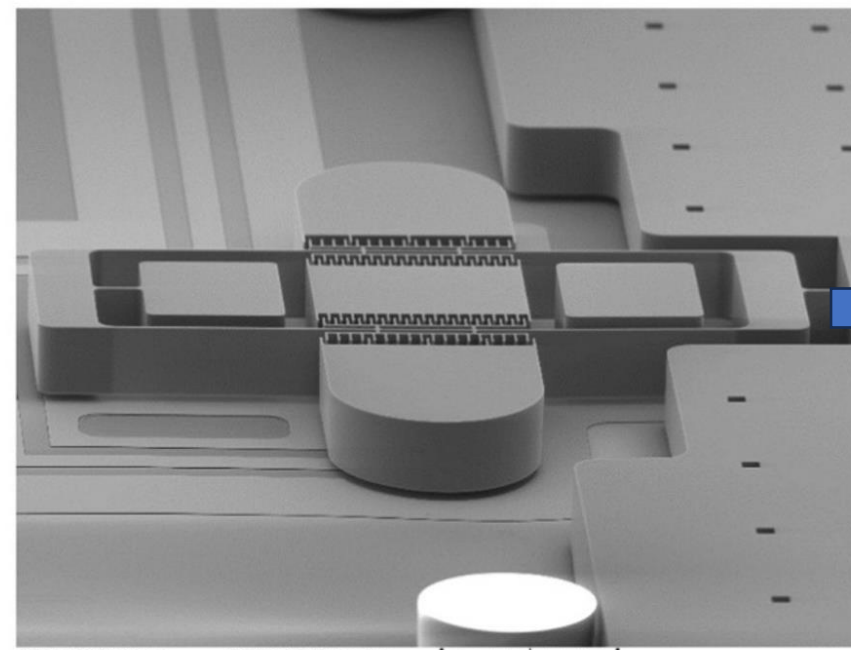
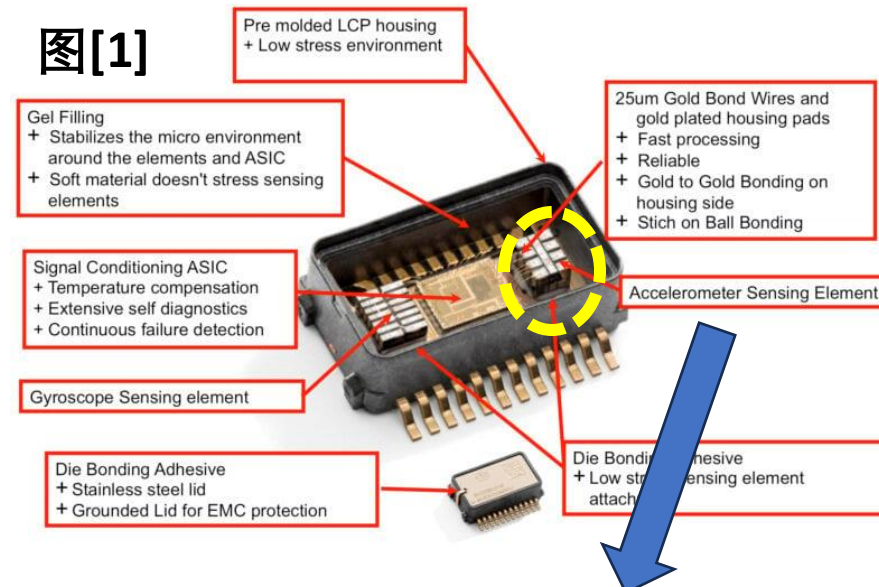
图[2]



SEM MAG: ---
HV: 20.0 kV
DET: SE Detector
VAC: HiVac

Vega ©Tescan

图[1]



SEM MAG: 234 x
HV: 20.0 kV
DET: SE Detector
VAC: HiVac

200 um

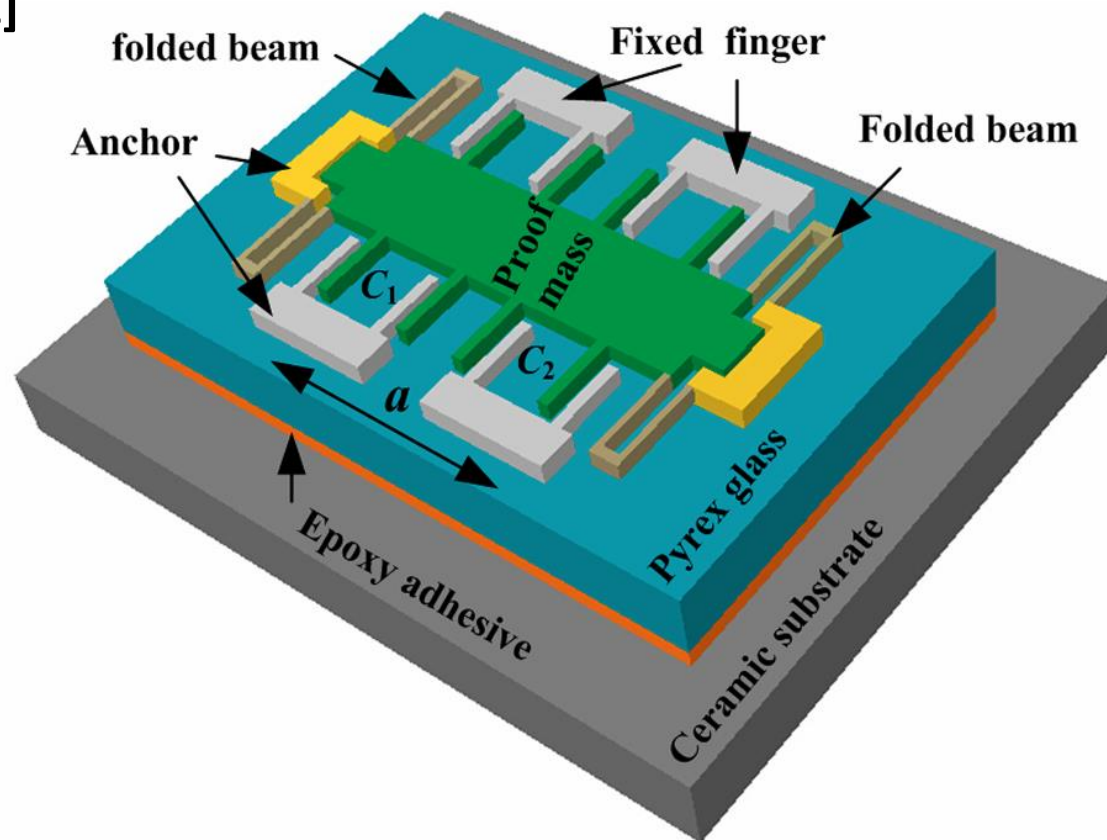
Vega ©Tescan

Capacitive
accelerometer
核心resonator

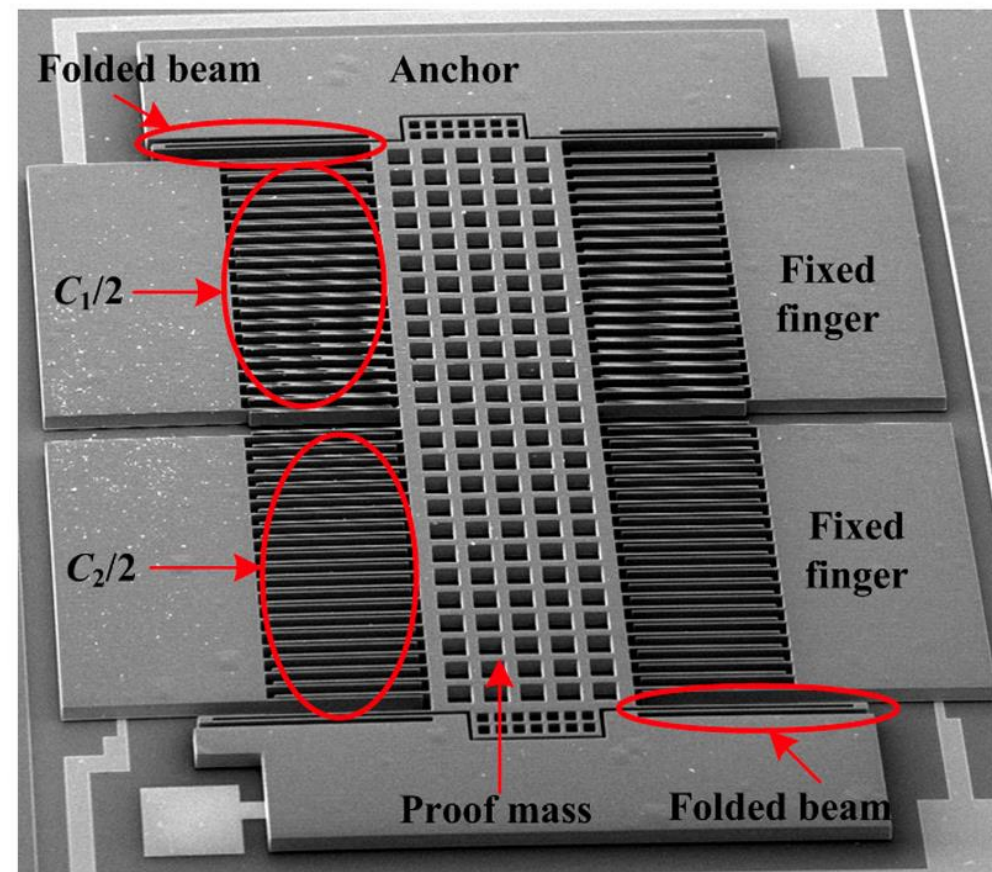


Capacitive accelerometer 核心resonator结构

图[1]



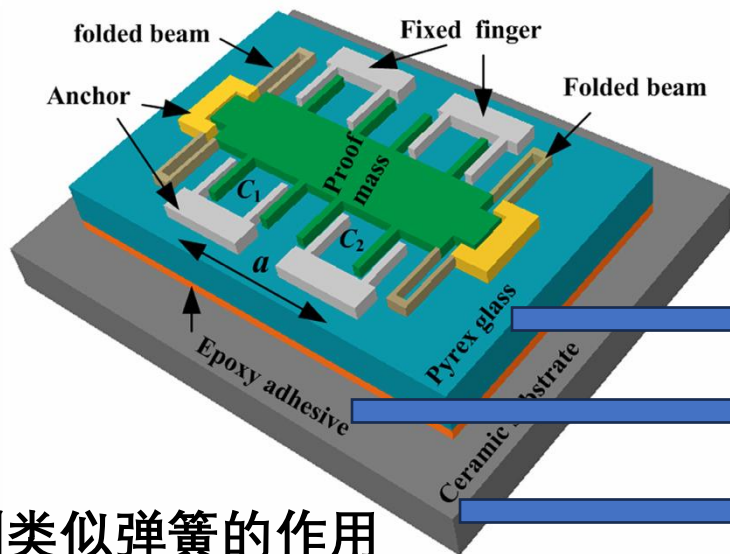
(a)



(b)

Figure 1. Structure of capacitive accelerometer. (a) Diagram of accelerometer; (b) SEM structure.

Capacitive accelerometer 核心resonator结构



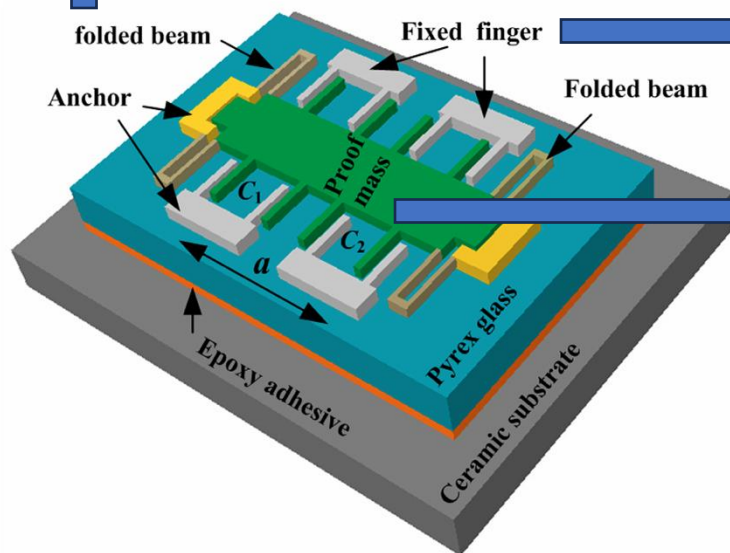
Pyrex glass (硼硅玻璃) 支撑结构之一，常作为盖层/绝缘层使用，提供支撑和封装环境

Epoxy adhesive (环氧胶)，用于把 MEMS 芯片粘接到封装基底（通常是陶瓷基板）上的胶粘材料。

Ceramic substrate (陶瓷基板)。整个传感芯片的基底

起到类似弹簧的作用

(a)



Fixed finger (固定电极)

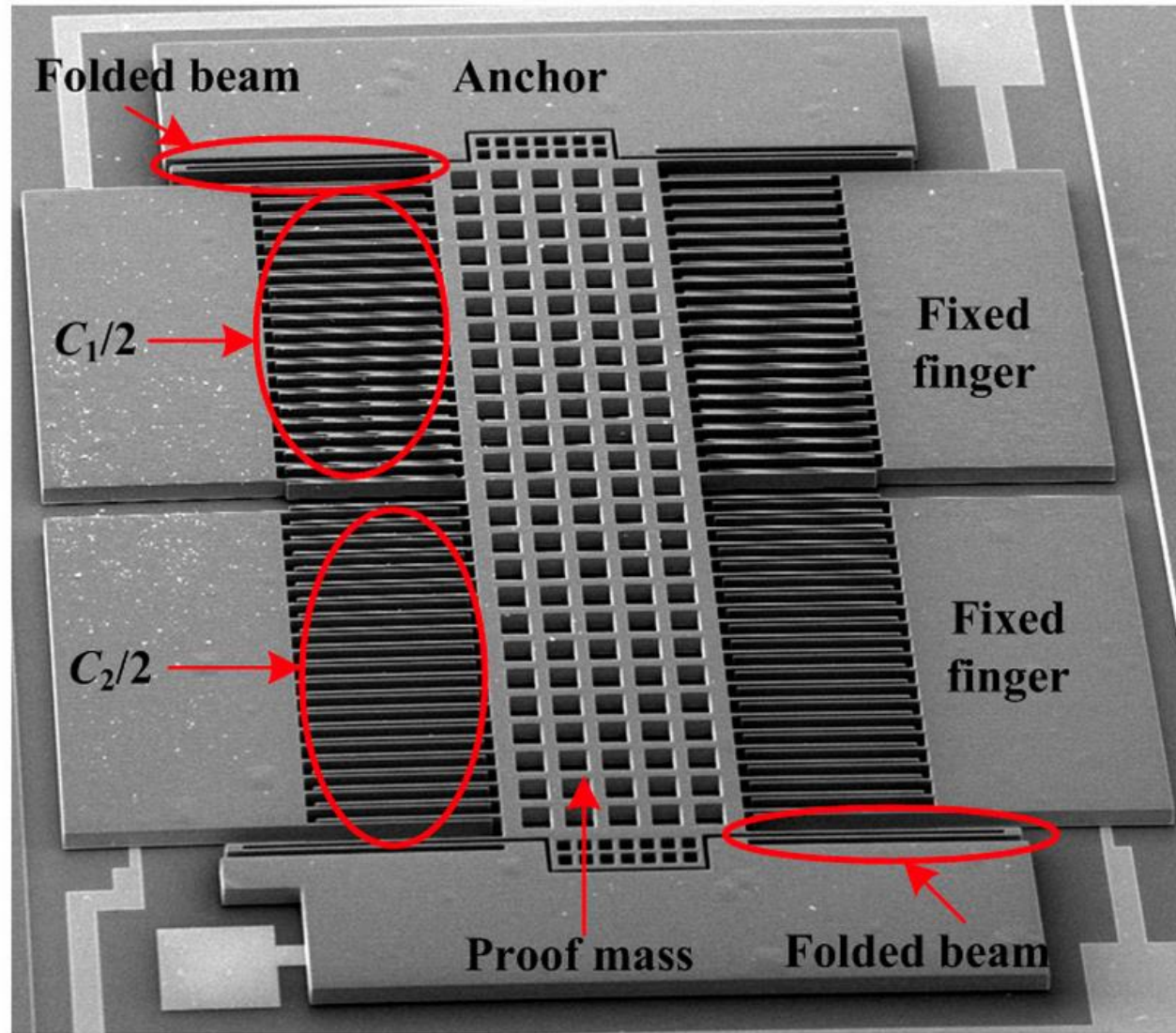
白色结构，是电容式加速度计的固定部分，与质量块上的移动电极一起构成差动电容器 (C_1 与 C_2)，**定梳齿**

Proof mass (质量块) 加速度计的核心感应元件。在受到加速度作用时，它会由于惯性而发生微小位移，引起电容变化。
也就是动梳齿

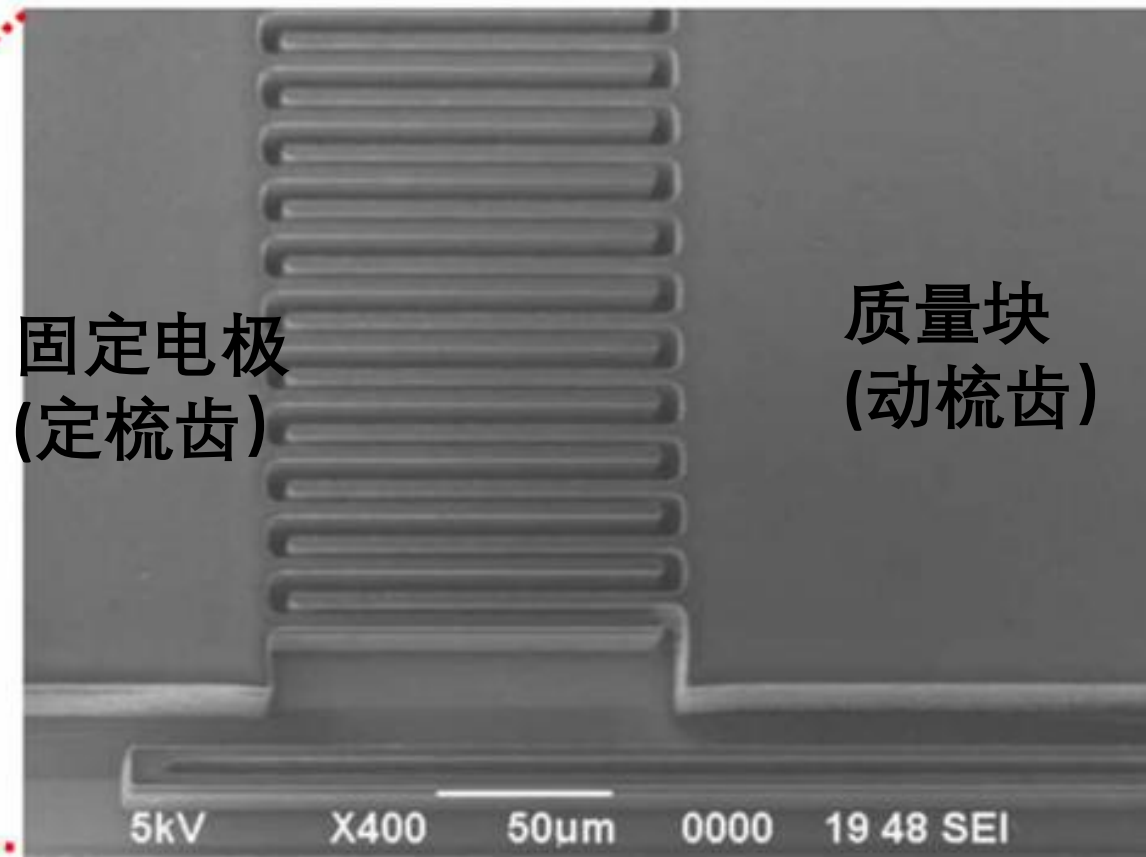
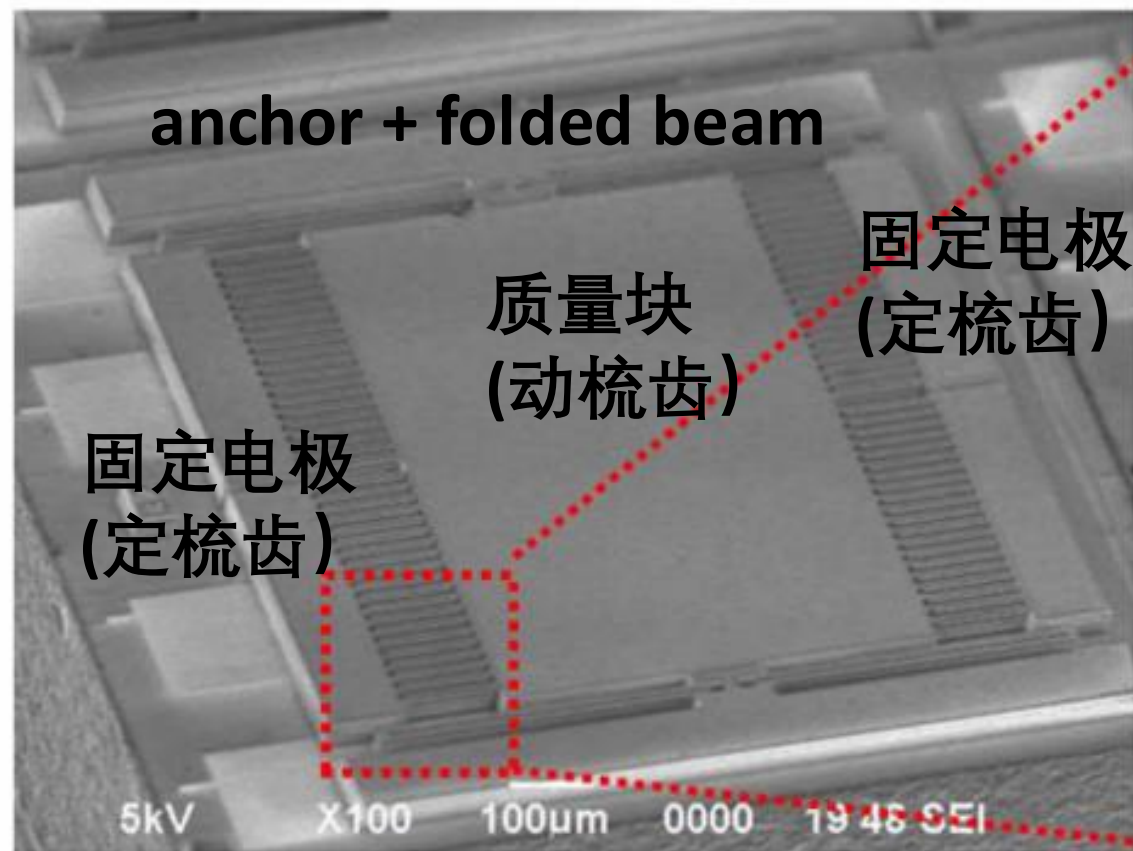
(a)



Capacitive accelerometer 核心resonator结构实际图

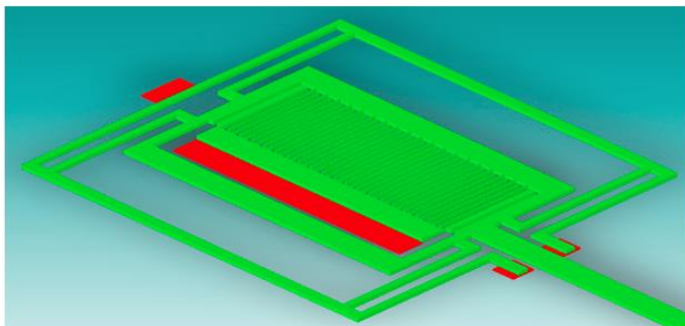


Capacitive accelerometer 核心resonator结构实际图

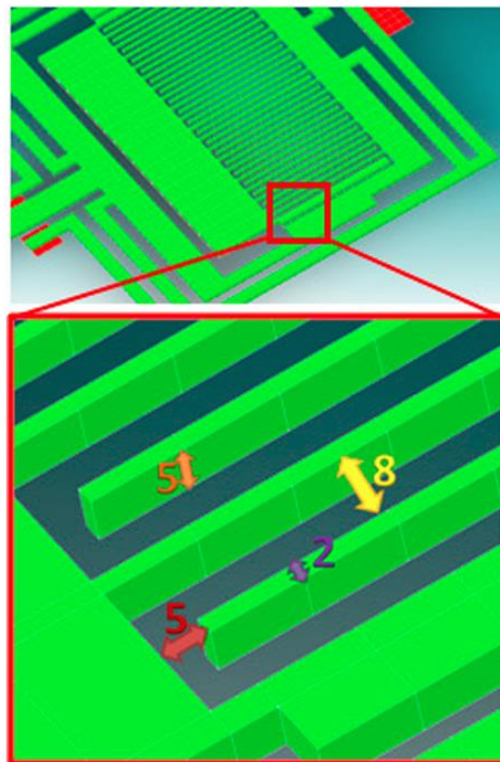




resonator结构实际尺寸



单位um



不同加速度下定梳齿移动距离

Table 2. The displacement of the seismic mass under acceleration.

Acceleration	Displacement
[m/s ²]	[nm]
0.5	0.054
1	0.120
1.5	0.160
2	0.220

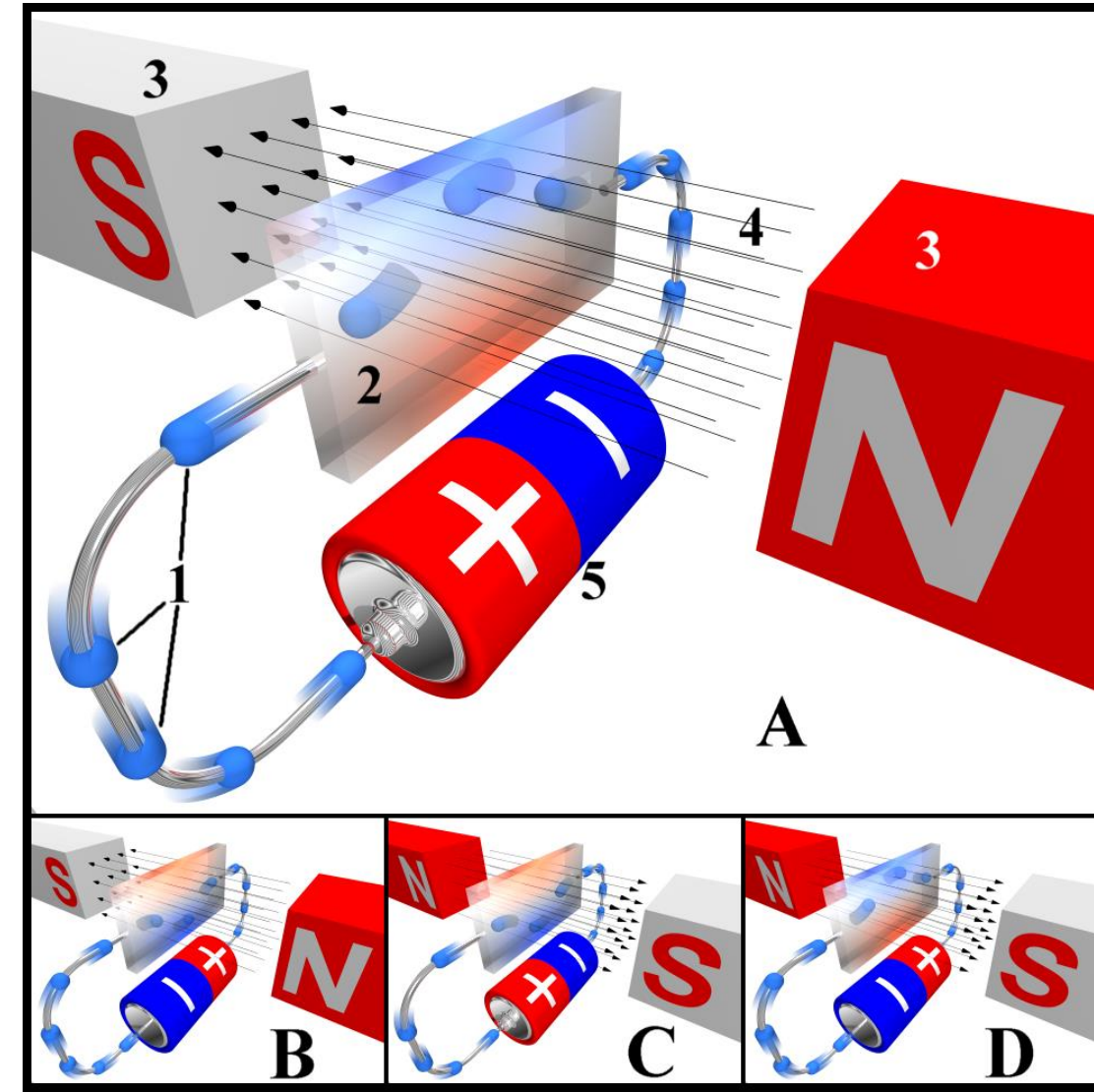


梳齿移动距离~nm,远小于
resonator电极板间隙



How Magnetometer Work

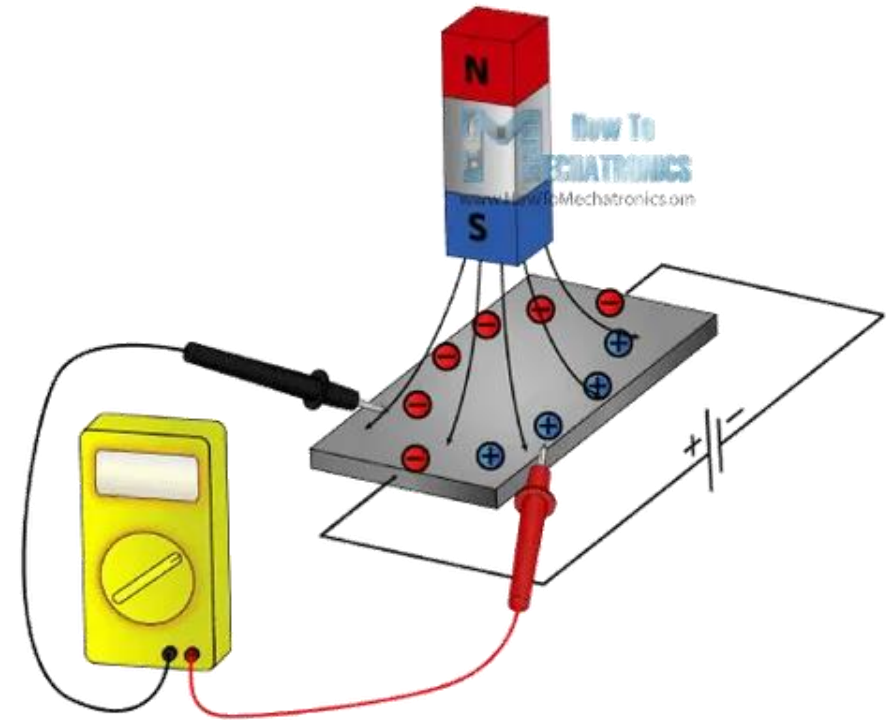
- Measures magnetic field strength (Tesla or Gauss)
- Foundation: Hall effect





How Magnetometer Work

- Measures magnetic field strength (Tesla or Gauss)
- Foundation: Hall effect + MEMS





Outline

- An Introduction of IMU
- **IMU Error Model**
- IMU Calibration



IMU Error Model





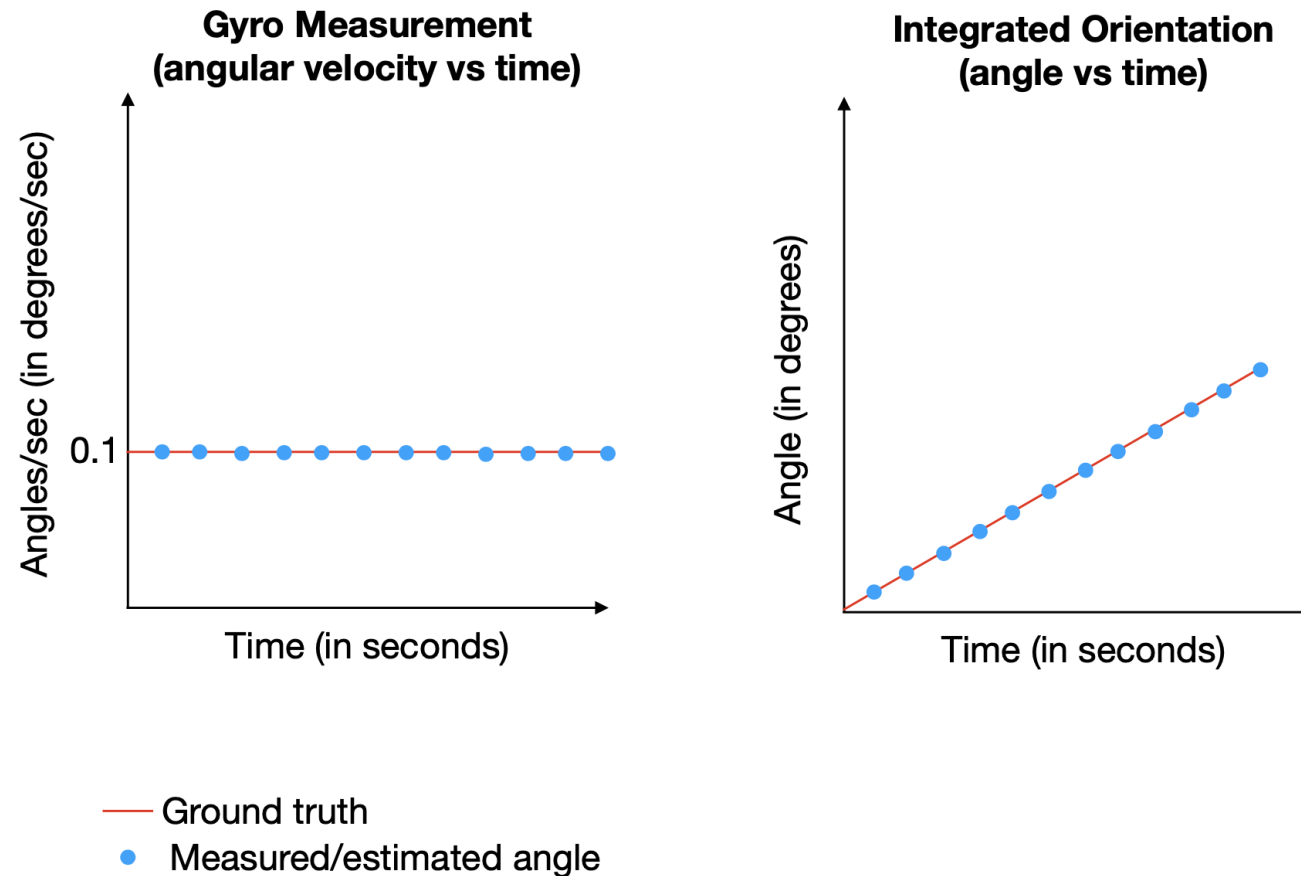
IMU Error Model

- Integration error for calculating position and angle
- Example
 - How to get from angular velocity to angle?
 - Integrate, knowing initial position



IMU Error Model: Angular Velocity $\rightarrow$ Angle

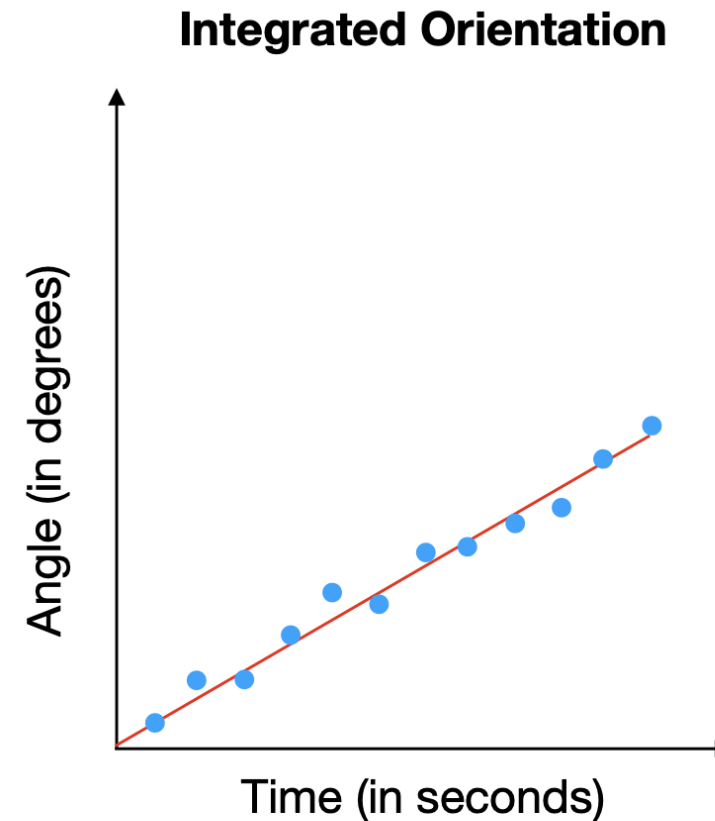
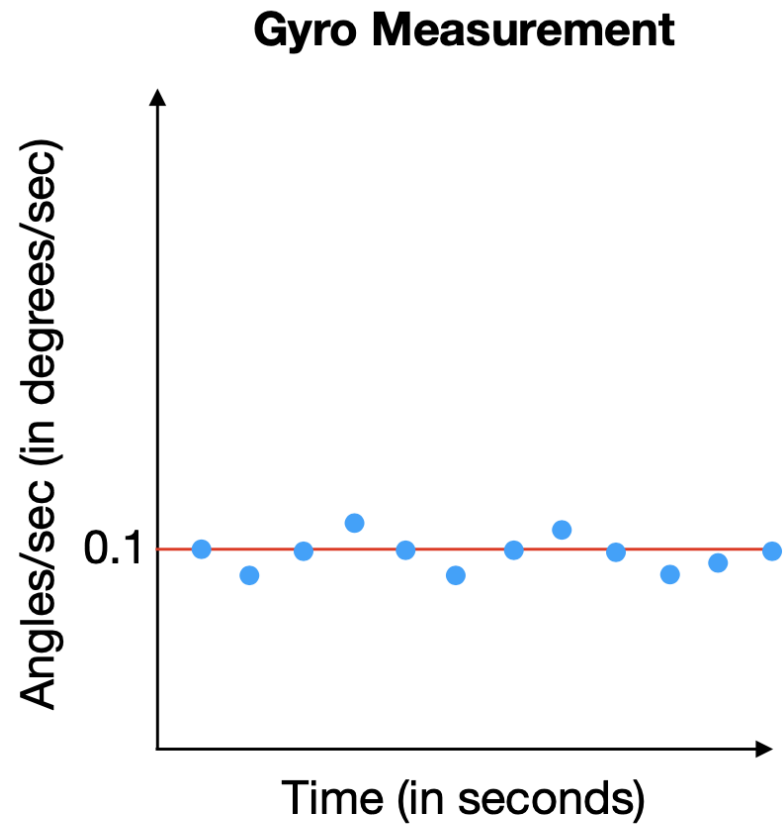
- no noise, no bias





IMU Error Model: Angular Velocity $\rightarrow$ Angle

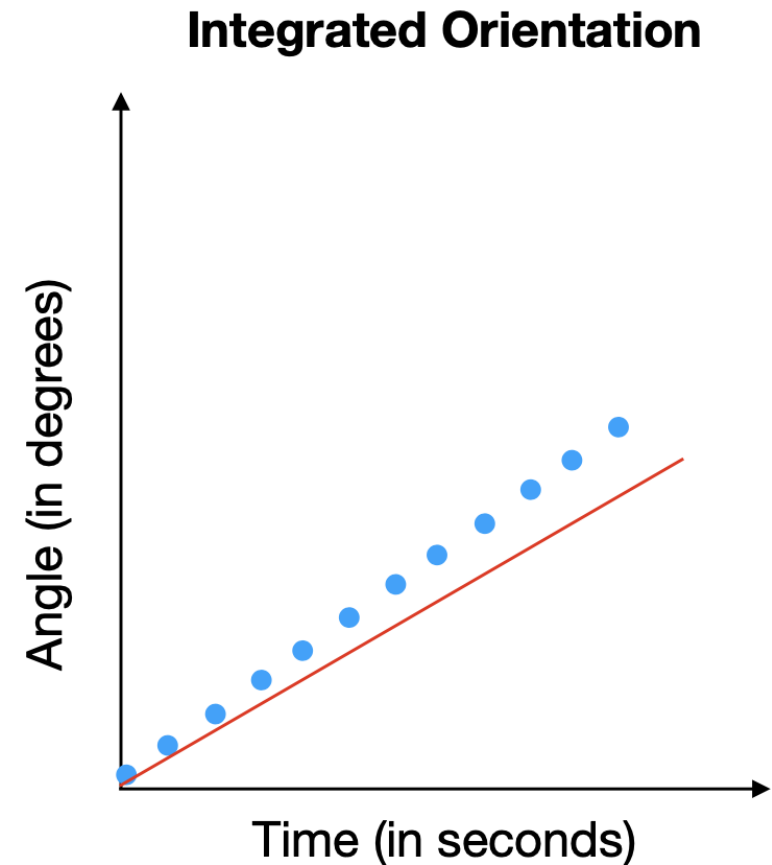
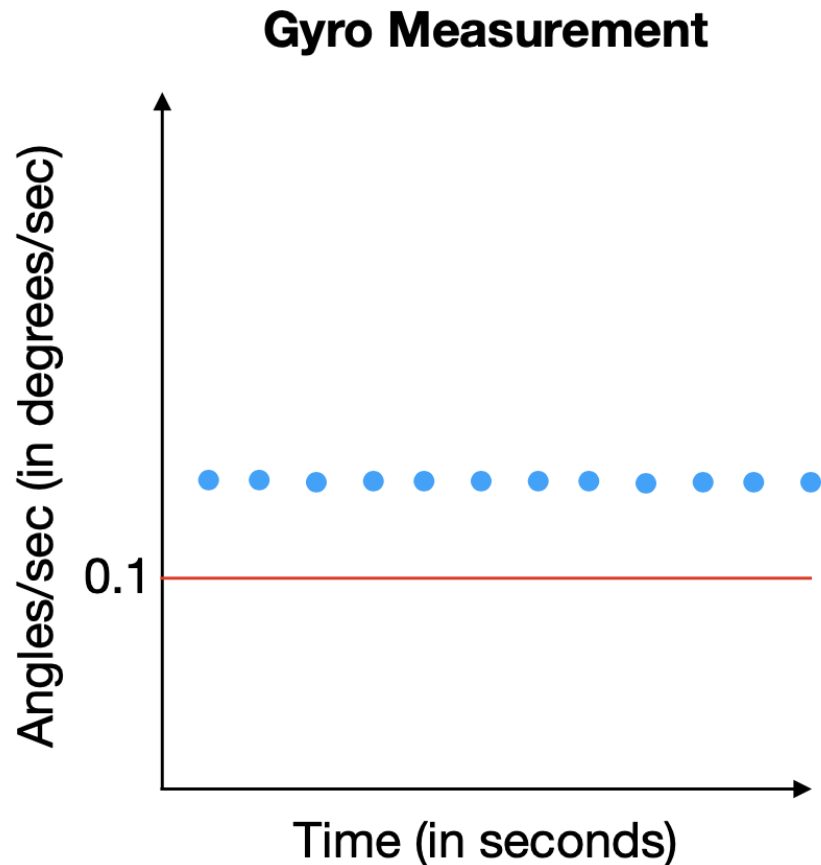
- noise, no bias





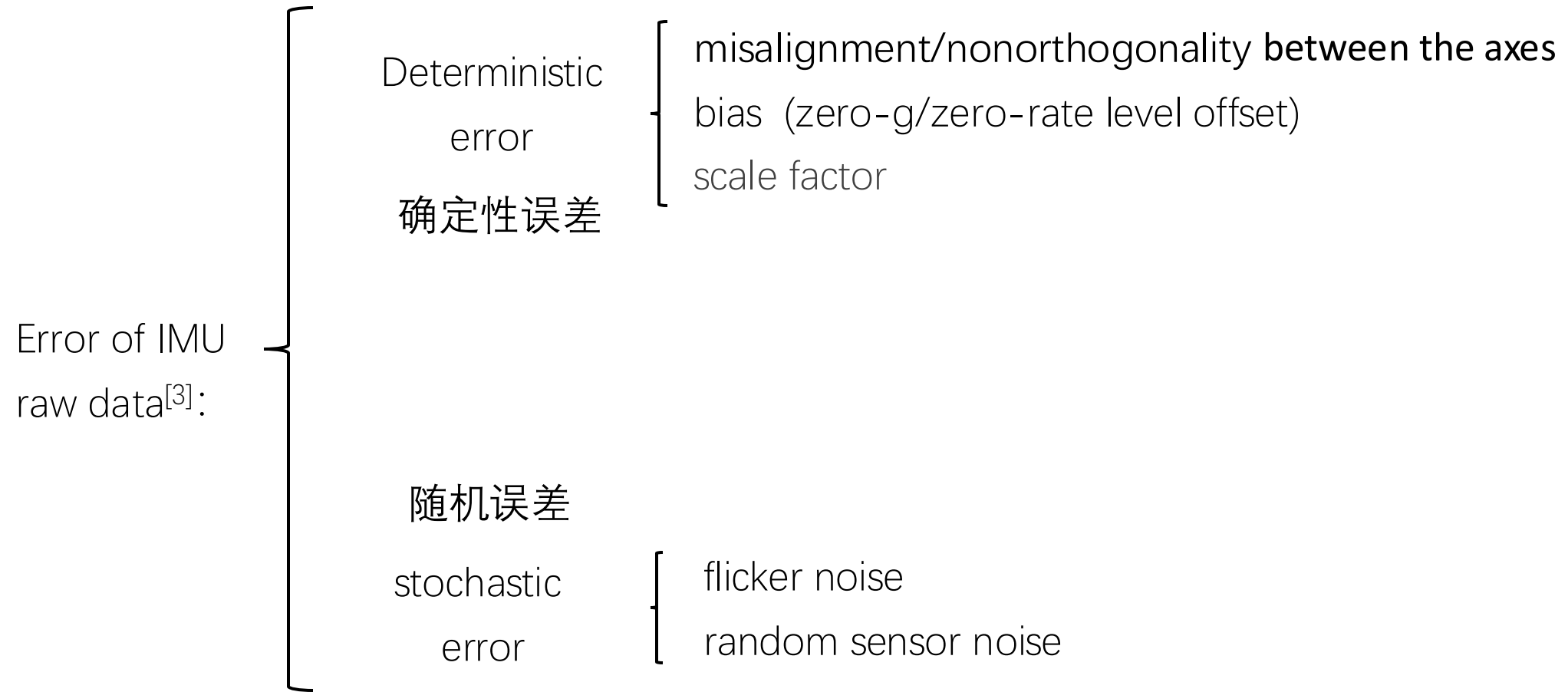
IMU Error Model: Angular Velocity $\rightarrow$ Angle

- no noise, bias





IMU Error Model



[3]D. Unsal and K. Demirbas, "Estimation of deterministic and stochastic IMU error parameters," *Proceedings of the 2012 IEEE/ION Position, Location and Navigation Symposium*, 2012, pp. 862-868, doi: 10.1109/PLANS.2012.6236828.

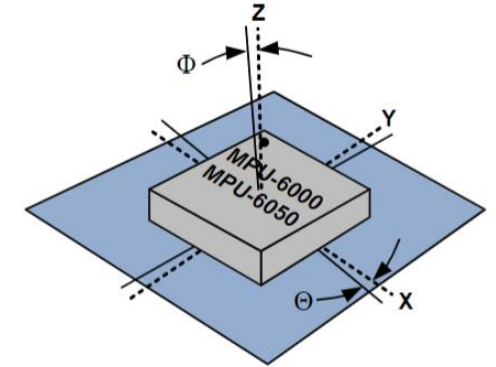


IMU Error Model

misalignment / nonorthogonality between the axes:

加速度、角速度坐标系不重合

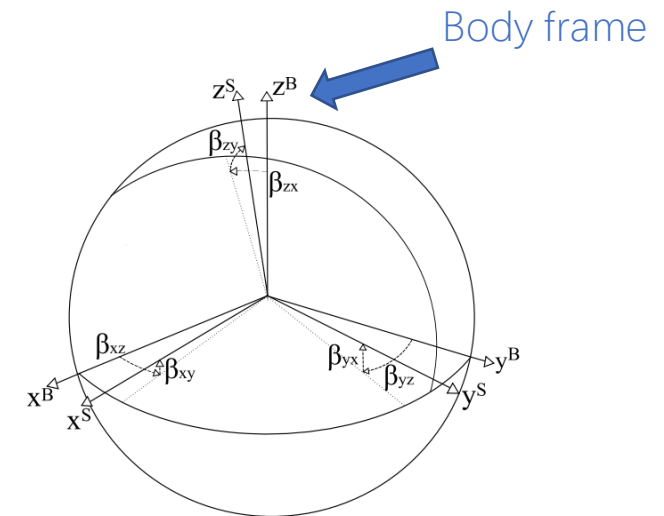
Root cause: mechanical components cannot be produced perfectly/PCB package



Calibration matrices

$$\mathbf{a}^O = \mathbf{T}^a \mathbf{a}^S, \quad \mathbf{T}^a = \begin{bmatrix} 1 & -\alpha_{yz} & \alpha_{zy} \\ 0 & 1 & -\alpha_{zx} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{\omega}^O = \mathbf{T}^g \boldsymbol{\omega}^S, \quad \mathbf{T}^g = \begin{bmatrix} 1 & -\gamma_{yz} & \gamma_{zy} \\ \gamma_{xz} & 1 & -\gamma_{zx} \\ -\gamma_{xy} & \gamma_{yx} & 1 \end{bmatrix}$$





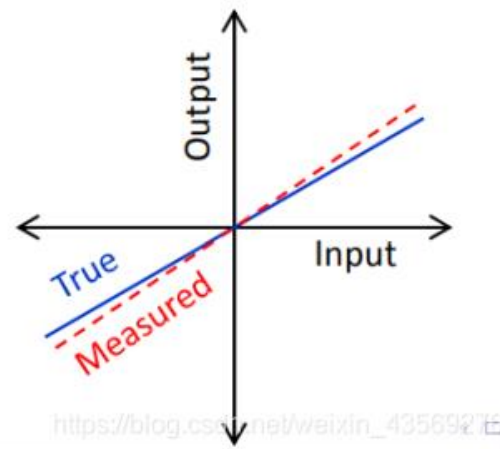
IMU Error Model

scale factor (存在一个比例因子)

Root Cause: introduced from A/D conversion

Calibration matrix: scaling matrices

$$\mathbf{K}^a = \begin{bmatrix} s_x^a & 0 & 0 \\ 0 & s_y^a & 0 \\ 0 & 0 & s_z^a \end{bmatrix}, \quad \mathbf{K}^g = \begin{bmatrix} s_x^g & 0 & 0 \\ 0 & s_y^g & 0 \\ 0 & 0 & s_z^g \end{bmatrix}$$



0g on the X- and Y-axes and +1g on the Z-axis. The accelerometers' scale factor is calibrated at the factory and is nominally independent of supply voltage. Each sensor has a dedicated sigma-delta ADC for providing digital outputs. The full scale range of the digital output can be adjusted to $\pm 2g$, $\pm 4g$, $\pm 8g$, or $\pm 16g$.

MPU6050 datasheet



IMU Error Mode

bias (zero-g/zero-rate level offset) + flicker noise(bias Instability):

零偏

- 闪烁噪声 (零偏不稳定性) : 零偏会随时间温度缓慢变化
- 即: 静止时, 衡量陀螺仪输出量围绕其均值 (零偏) 的离散程度

Root Cause: quite complex, e.g., sensitive to environment/temperature

Calibration: two bias vectors

$$\mathbf{b}^a = \begin{bmatrix} b_x^a \\ b_y^a \\ b_z^a \end{bmatrix}, \quad \mathbf{b}^g = \begin{bmatrix} b_x^g \\ b_y^g \\ b_z^g \end{bmatrix}$$

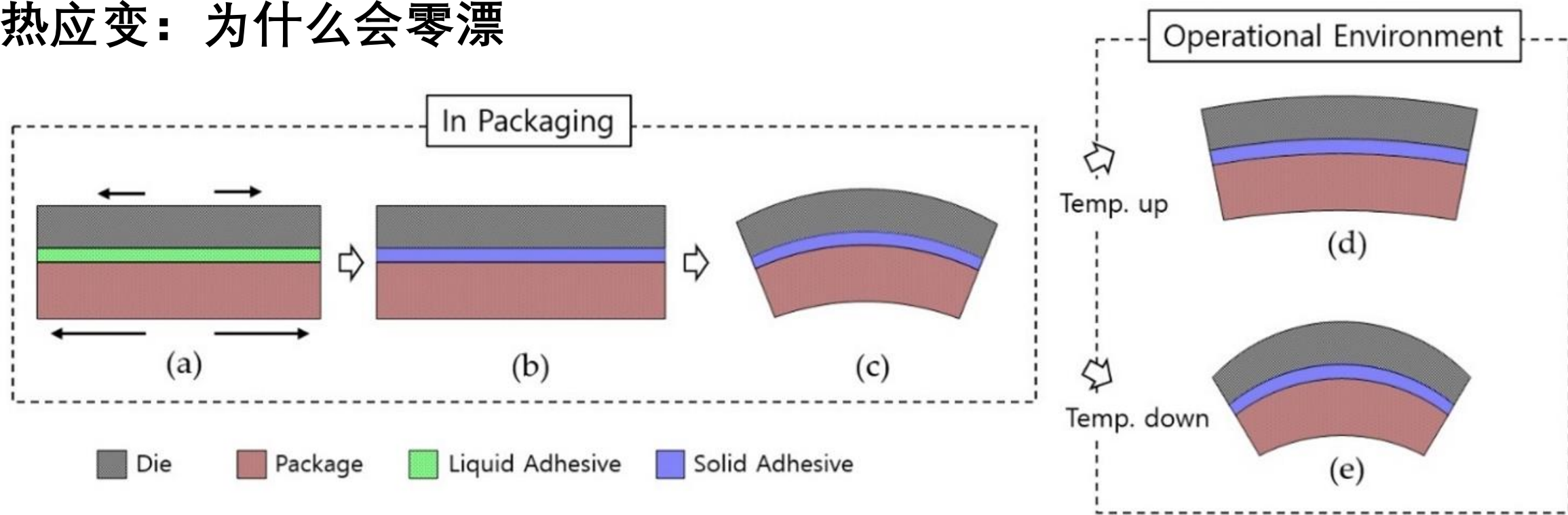
Note:

- The bias is different every time you turn an IMU on due to flicker noise
- The bias is different even if the IMU is operating after a long time

Definition of application grades		
Gyroscope	In-Run (for industrial / tactical) or day to day (for navigation / strategic)	Corresponding Grade
Bias Stability		
100°/h		Industrial
5°/h		
1°/h		Tactical
0.5°/h		
0.1°/h		Short term Navigation
0.05°/h		
0.01°/h		High-end navigation & Strategic
0.0001°/h		



热应变：为什么会零漂



芯片粘接（die attach）：通常也是产生热应力的主要来源

通常，MEMS 传感器的封装材料(Package)的热膨胀系数要大于芯片材料(Die)

在封装过程中，当温度升高至胶水固化温度之前，封装体(Package)比芯片(Die)膨胀得更多（图 1a）

此时，封装体和芯片已膨胀到不同程度，随后在固化温度下用胶水固定在一起（图 1b）

在随后的冷却过程中，由于封装体与芯片的收缩速率不同，会产生热应力，使得整个结构发生变形，向下凹陷（图 1c）

在工作环境中，这种称为“芯片翘曲（die warpage）或封装应力（packaging stress）”的变形，会随温度变化而波动：温度升高时减小，温度降低时增强（图 1d、1e）



IMU Error Model

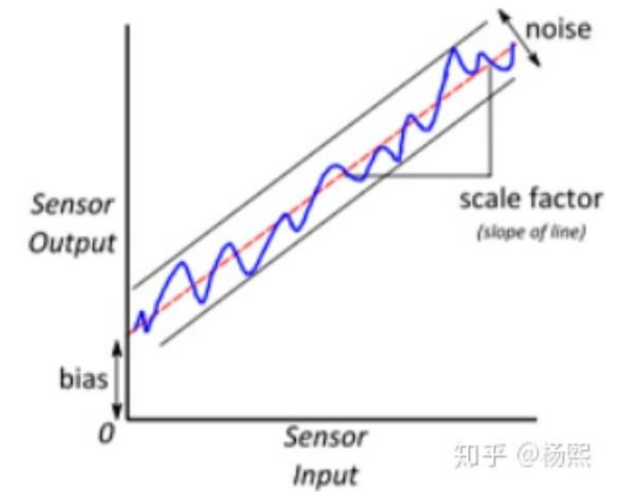
random sensor noise

Root Cause: sensitive to environment/temperature/pressure.

How to express in calibration model:



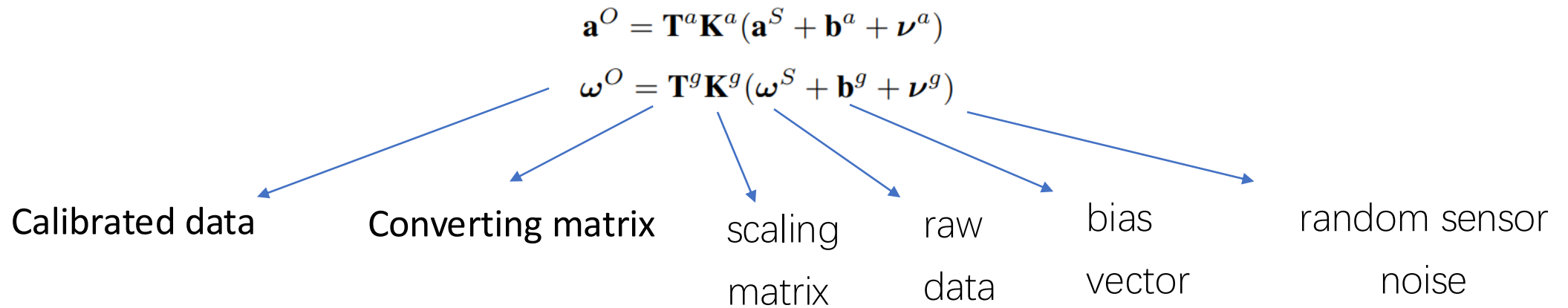
ν^g





IMU Error Model: In Summary

Complete sensor error model:



For low-precision use:

Only need to evaluate the bias of IMU

For relative high-precision use:

Each error parameter should be carefully evaluated



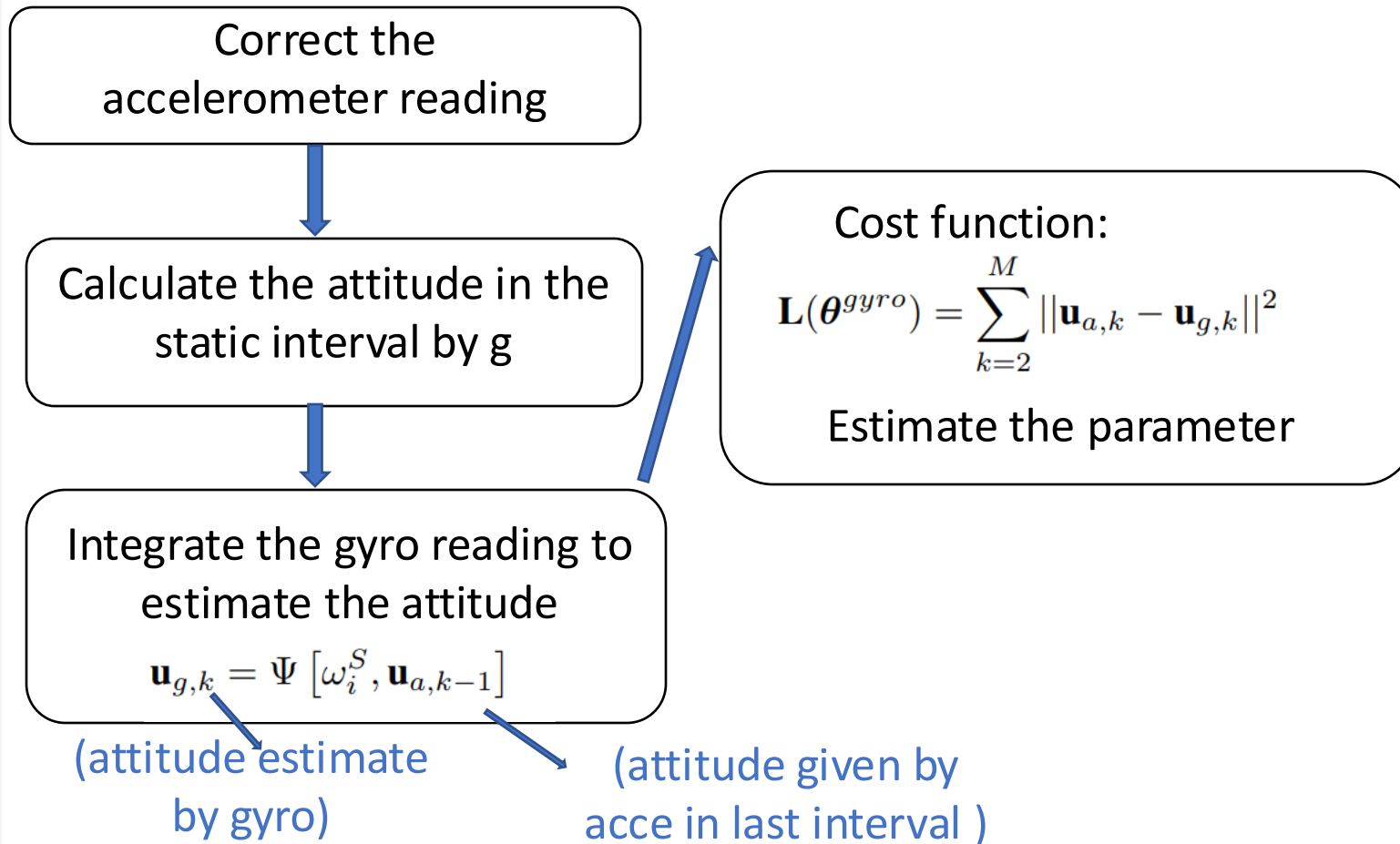
Outline

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- IMU Error Model
- IMU Calibration



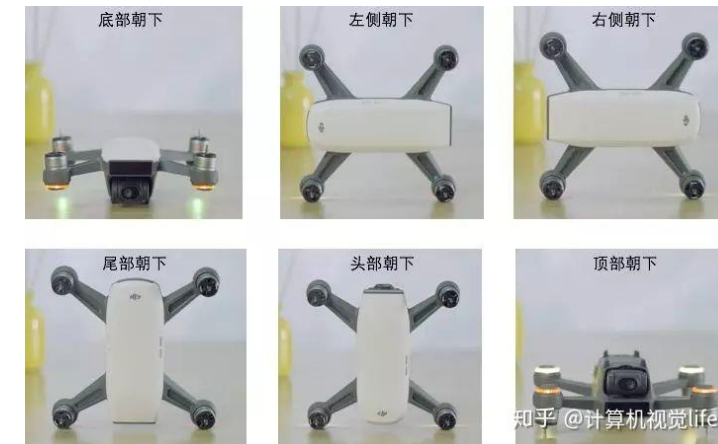
IMU Calibration

Use the calibrated accelerometer to calibrate gyroscope:



Other calibration method:

Six - sided calibration method



High precision turntable

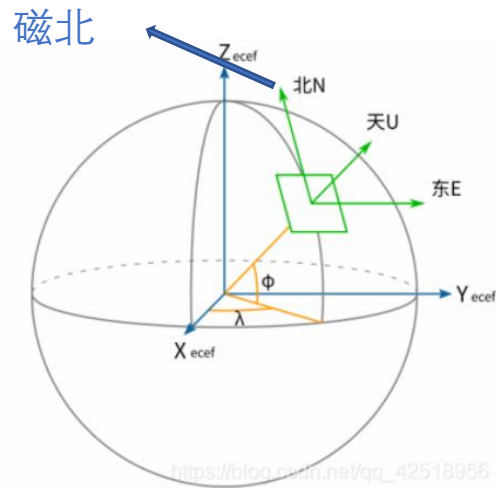




IMU for Altitude Estimation

Something need to know before estimating attitude:

1、Earth coordinate(ENU)



The attitude refers to the change between ENU and IMU's own coordinate(body frame)

2、Euler angle $\rightarrow$ Expression
quaternion(四元数) $\rightarrow$ calculation



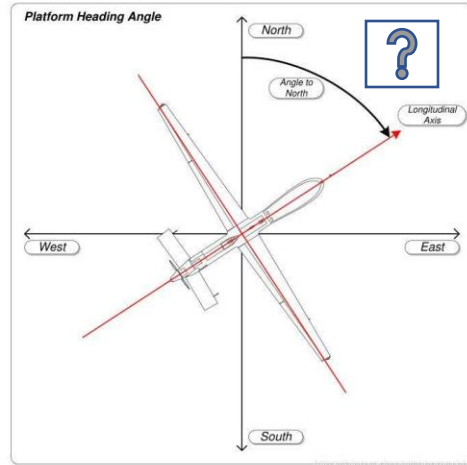
Euler angle:

Yaw/Pitch/Roll(Z-Y-X)



IMU for Altitude Estimation

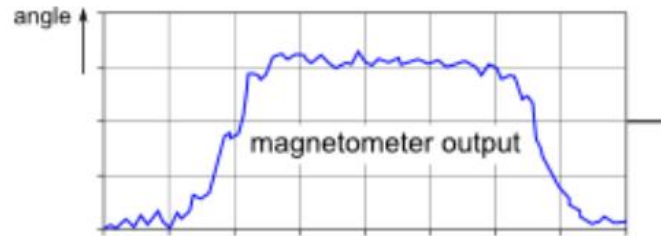
accelerometer :



Use gravity to calibrate:

- 1、 IMU keep moving:
acceleration= $a + g$
- 2、 difficult to calculate yaw angle

magnetometer:



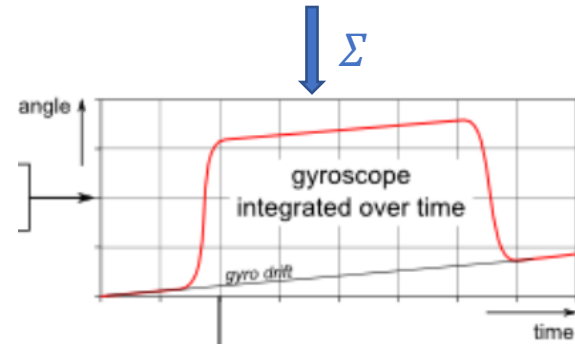
Use earth magnetic field to calibrate:

- 1、 suffer from high frequency noise, calculated pose will oscillate
- 2、 sensitive to magnetic interference

gyroscope:



Suffer from
deterministic
error/noise



Use integration of angular rate:
drift problem



Crash detection

- https://www.bilibili.com/video/BV1Be4y1h7pe/?spm_id_from=333.337.search-card.all.click
 - (1:32:18 – 1:32:48)





References

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